

The Farmer's Stone

To weigh bales of hay, Farmer Jones has a balance beam scale and a stone weighing exactly 40 pounds. One day, his neighbor Farmer Smith borrowed the scale and the stone. When he returned, he was very apologetic. "The stone fell out of my wagon and broke into four pieces", he said.



A week later, Farmer Jones saw Farmer Smith and said, "You actually did me a favor by breaking that stone. Now, I can exactly weigh any amount of hay in integer pounds from 1 to 40." What are the weights of the four pieces of stone?

Solution to The Farmer's Stone

The weights of the four pieces of stone are 1, 3, 9, and 27 pounds. The table below shows how these four pieces and a balance beam scale can be used to measure any integer number of pounds from 1 to 40.

It is not a coincidence that the weights of the four pieces are the first four integer powers of 3. If a fifth piece of stone weighing 81 pounds were added, any integer number of pounds from 1 to 121 could be weighed (extending the table is left as a homework exercise).

To weigh 1 1 = 1	To weigh 21 21+9 = 3+27
To weigh 2 2+1 = 3	To weigh 22 22+9 = 1+3+27
To weigh 3 3 = 3	To weigh 23 23+1+3 = 27
To weigh 4 4 = 1+3	To weigh 24 24+3 = 27
To weigh 5 5+1+3 = 9	To weigh 25 25+3 = 1+27
To weigh 6 6+3 = 9	To weigh 26 26+1 = 27
To weigh 7 7+3 = 1+9	To weigh 27 27 = 27
To weigh 8 8+1 = 9	To weigh 28 28 = 1+27
To weigh 9 9 = 9	To weigh 29 29+1 = 3+27
To weigh 10 10 = 1+9	To weigh 30 30 = 3+27
To weigh 11 11+1 = 3+9	To weigh 31 31 = 1+3+27
To weigh 12 12 = 3+9	To weigh 32 32+1+3 = 9+27
To weigh 13 13 = 1+3+9	To weigh 33 33+3 = 9+27
To weigh 14 14+1+3+9 = 27	To weigh 34 34+3 = 1+9+27
To weigh 15 15+3+9 = 27	To weigh 35 35+1 = 9+27
To weigh 16 16+3+9 = 1+27	To weigh 36 36 = 9+27
To weigh 17 17+1+9 = 27	To weigh 37 37 = 1+9+27
To weigh 18 18+9 = 27	To weigh 38 38+1 = 3+9+27
To weigh 19 19+9 = 1+27	To weigh 39 39 = 3+9+27
To weigh 20 20+1+9 = 3+27	To weigh 40 40 = 1+3+9+27