

Boarding the Plane

You are the last of 100 passengers in line to board a 100-seat airplane. Everyone in line has a boarding pass with a reserved seat number on it. However, the first guy in line drops his boarding pass, so he takes a seat chosen at random. As the other passengers board the plane, each takes his or her assigned seat unless someone is already sitting in it. In that case, he or she takes an open seat chosen at random. What is the probability that you will wind up in your assigned seat?

Solution to Boarding the Plane

For convenience in the following explanation, let us label each passenger with his or her position in line – the first person in line is Passenger 1, the next is Passenger 2, etc. Let us then label each seat on the plane by the number of the passenger assigned to it – so, for example, Passenger 20 is assigned to Seat 20.

It is *always* true that, for $N = 2$ to 99, after Passenger N has boarded, Seat N is occupied. (When Passenger N boards, either he or she takes Seat N or it is already occupied.) So, when Passenger 100 boards, the last remaining open seat must be either Seat 1 or Seat 100. Whichever of those two seats is already occupied was taken by someone (including, possibly, Passenger 1) who was choosing randomly among the open seats. In each random choice, every open seat has the same probability of being taken. So, it is equally likely that Seat 1 or Seat 100 will be taken by the time that Passenger 100 boards. The probability that Passenger 100 gets his or her assigned seat is 50%.

This answer can also be found via simulation.