SDP

DEC 2022 BRAIN TEASER & SOLUTION

Table Tennis Tournament

Three friends, tennis table champions long ago, have a family reunion. Reminiscing their glory days, they challenge each other to a tournament for bragging rights. Denoting the friends as A, B and C, the first game is played by A and B, while C rests. Thereafter, the game loser sits out and the resting player takes on the prior game winner. Whomever wins two games in a row wins the tournament.

Question 1: They agree to a winner's plaque with a record of the games played. The tournament is recorded by listing in order the winner of each game, for example ACC records a 3-game tournament won by C, with the first game won by A. Which of the following sequences are plausible tournament outcomes?

a) ACB; b) ABB; c) ACAA; d) ACBB; e) BCABB; f) BCBCAA

Question 2: The spouses are concerned they only have two hours of available time, with each game on average lasting 30 minutes. Determine the probabilities of each player A, B and C winning the tournament in 4 games or less, and the chance that after 4 games the tournament is undecided. Each player is of equal strength and just as likely to win any game.

Question 3: Same as 2, but A and B are of equal strength, while C is stronger and likely to win 60% of games versus both A and B.

Note: Credit to Keith McNulty and Cambridge University Sixth Term Examination Papers (UK) for the original problem, after which this problem was developed using the same tournament structure.

The answer to the Dec 2022 Brain Teaser - Table Tennis Tournament

Question 1

a) ACB; Not plausible - no winner of two games in a row

b) ABB; Not plausible - after A wins first game the second game is A vs C, so B cannot win second game.

c) ACAA; Not plausible - after C wins second game the third game is C vs B, so A cannot win third game.

d) ACBB; Yes plausible.

e) BCABB; Yes plausible.

f) BCBCAA; Not plausible - after C wins second game the third game is C vs A, so B cannot win third game.

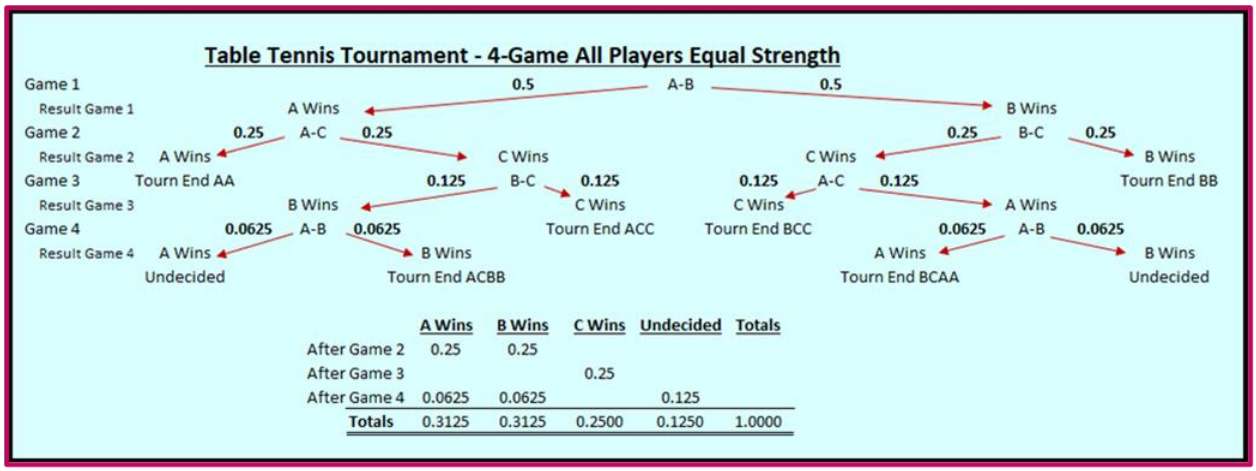
Question 2

Probabilities: A wins 31.25%; B wins 31.25%; C wins 25%; tournament is undecided 12.5%.

Question 3

Probabilities: A wins 26%; B wins 26%; C wins 36%; tournament is undecided 12%.

Questions 2 and 3 require the construction of a decision tree like the one below. The tree shown below is for Q2, whereas for Q3 the probs need to be adjusted 60%-40% in favor of Player C.



	A Wins	B Wins	C Wins	Undecided	Totals
After Game 2	0.25	0.25			
After Game 3			0.25		
After Game 4	0.0625	0.0625		0.125	
Totals	0.3125	0.3125	0.2500	0.1250	1.0000