Splash-Out

Three contestants A, B, and C are competing in a TV game show called Splash-Out. The three are standing at the corners of a triangle exactly 25 feet from each other. The show's MC will hand to each contestant in turn a water balloon, which that contestant may throw at one of the others. A contestant is knocked out of the game when hit by a water balloon. The game continues until only one contestant remains; that contestant wins a valuable prize.

The three contestants have vastly different levels of throwing skill. Contestant A is by far the worst - on any given throw, there is only a one-third chance that A will hit his target. Contestant B is much better - on any given throw, there is a two-thirds chance that B will hit his target. And Contestant C is a major league baseball pitcher - C is certain to hit his target on every throw.

The MC has declared that the contestants will be given water balloons in the following order: A,B,C (repeating the sequence as necessary).

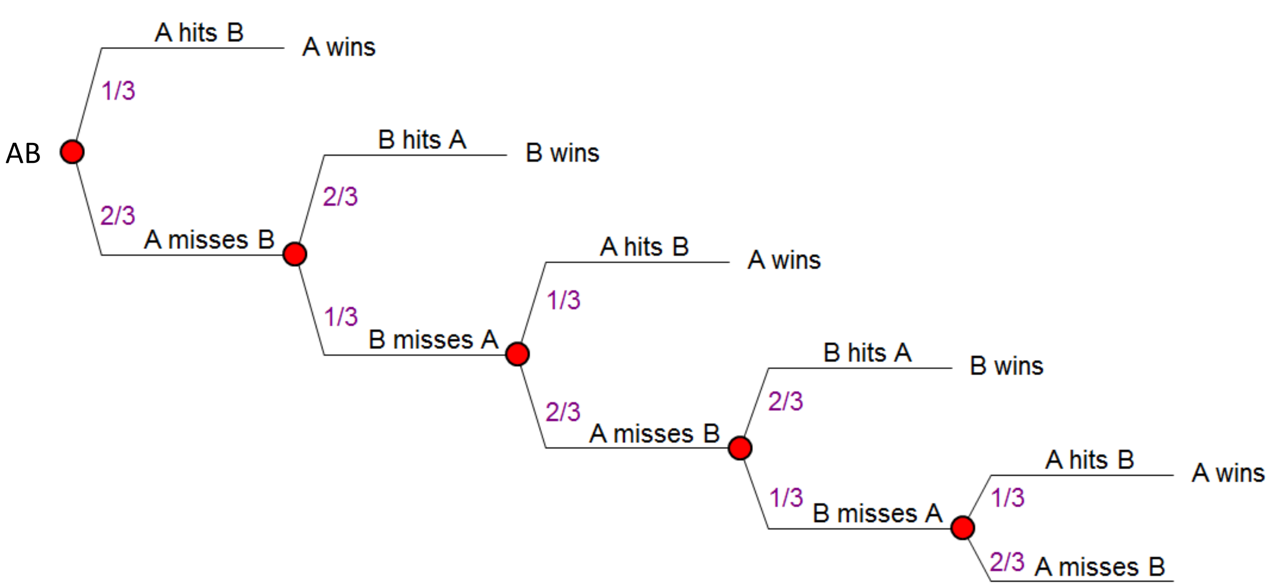
Assuming that each contestant acts to maximize his own chance of winning the prize and that they all agree with the probabilities stated above, what is the probability that each contestant will win the prize?

Solution to Splash-Out

Let us use the following notation to represent the current state of the game: "ABC" means that all three contestants are still in the game and that A has the next throw; "CA" means that only contestants C and A remain in the game and that C has the next throw; and so forth.

Let us first consider the winning probabilities for the 2-contestant states of the game.

CA – sure win for C (0, 0, 100%)
CB – sure win for C (0, 0, 100%)
AC – A has a 1/3 chance of winning, otherwise C wins (33%, 0, 67%)
BC – B has a 2/3 chance of winning, otherwise C wins (0, 67%, 33%)
AB – Consider the infinite probability tree shown below



From this tree, we can see that the probability that A wins is given by an infinite series:

Prob. that A wins $= (1/3) + (2/3)(1/3)(1/3) + (2/3)(1/3)(2/3)(1/3)(1/3) + \ldots\ldots\ldots$

$= (1/3)\,[1 + (2/9) + (2/9)^2 + (2/9)^3 + \ldots\ldots]$

$= (1/3)/(1 - 2/9)$

$= (3/7) = 43\%$

So, AB – (43%, 57%, 0)

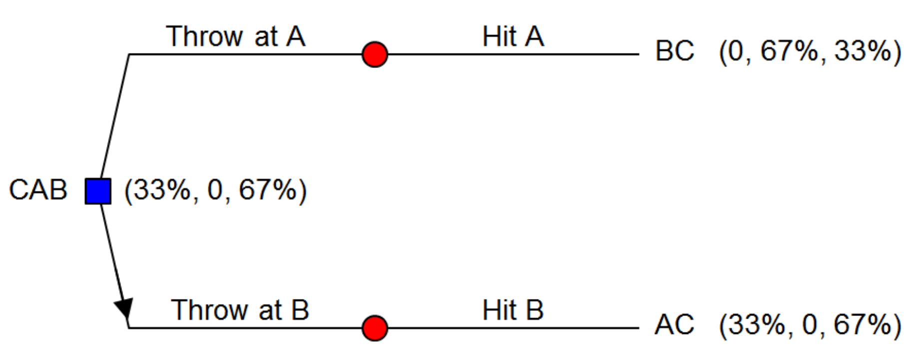
BA – For A to have a chance to win in this state, B would need to miss (p = 1/3), leading to state AB. So, A's probability of winning is (1/3) x (3/7) = 1/7 = 14%

So (14%, 86%, 0)

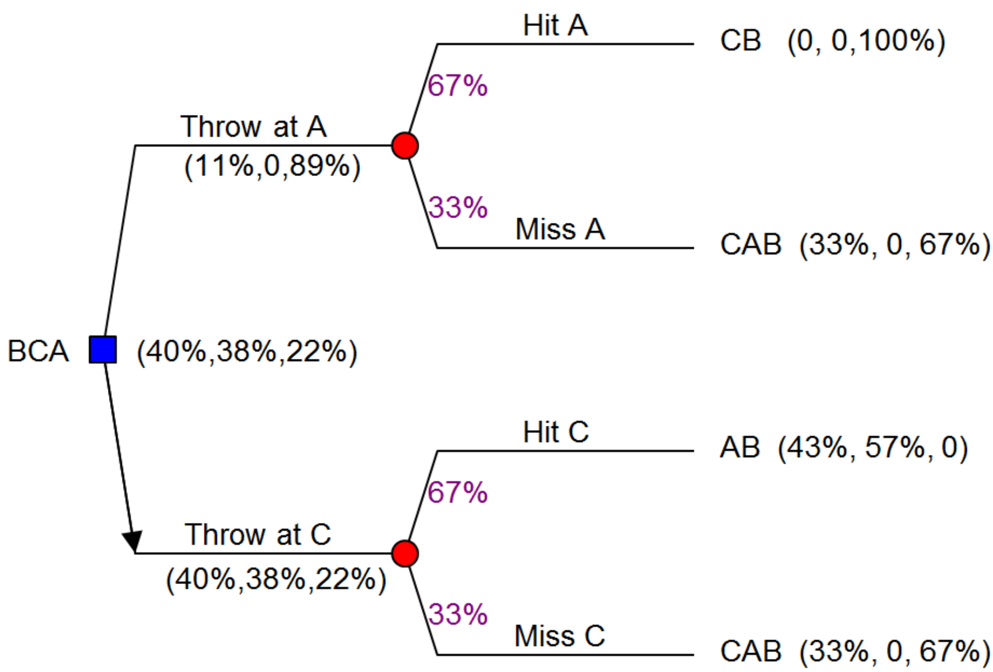
To summarize the 2-contestant states

- CA – (0, 0, 100%)
- CB – (0, 0, 100%)
- AC – (33%, 0, 67%)
- BC – (0, 67%, 33%)
- AB – (43%, 57%, 0)
- BA – (14%, 86%, 0)

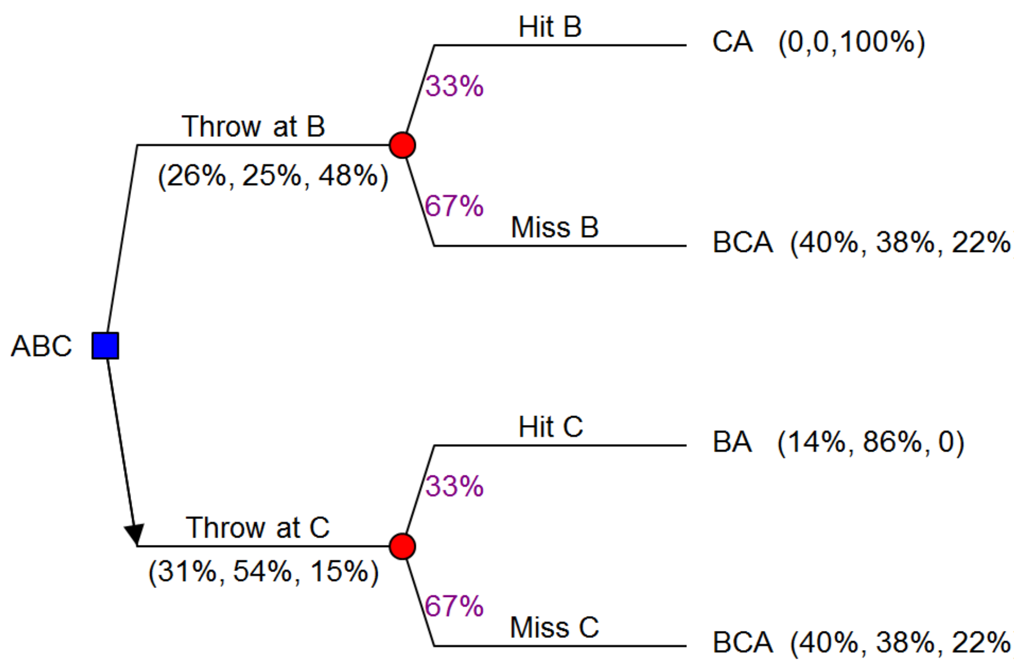
Let us now consider state CAB. As shown in the tree below, C has a choice to make – throw at A or throw at B. If he throws at A, he will hit him for sure and wind up in state BC, which we have seen gives C a 33% chance of winning. On the other hand, if C throws and hits B, he winds up in state AC in which C has a 67% chance of winning. So, to maximize his chance of winning, he will throw at and eliminate B. The probabilities for state CAB are then (33%, 0, 67%).



The situation for state BCA is shown in the tree below. If B chooses to throw at A, there is a 67% chance that he will hit A, creating state CB, and a 33% chance that he will miss, creating state CAB. Note that, in either case, B's chance of winning are zero. On the other hand, if B chooses to throw at C, there is a 67% chance that he will hit him, setting up state AB in which, as we have seen, B has a 57% chance of winning. So, throwing at C is B's better choice, giving him a 38% chance of winning. Thus, the probabilities for state BCA are (40%, 38%, 22%).



Finally, consider the state at the start of the game, ABC. The tree below shows that if A chooses to throw at B, there is a 33% chance that he will hit B, setting up state CA in which C wins for sure, and a 67% chance that he will miss B, setting up state BCA in which A has a 40% chance of winning. Overall, then, A has a 26% chance of winning if he throws at B. If he chooses to throw at C, there is a 33% chance that he will hit C, setting up state BA in which he has a 14% chance of winning, and a 67% chance of missing C, setting up state BCA in which he has a 40% chance of winning. Overall, his chance of winning is 31% if he throws at C.



But there is a very interesting insight hidden in the tree above. Notice that when A throws at either B or C, he will hope that he misses because his chance of winning is <u>lower</u> if he hits his target than if he misses. That suggests a better alternative for A – simply throw the water balloon into the ground, setting up state BCA for sure. So, the overall probabilities for the Splash-Out game are those for state BCA – (40%, 38%, 22%).