Cleaner Analysis:Quicker Decisions

Three Examples from Government

Decision Analysis Affinity Group

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Introduction

- **Technical Decisions for Government Projects Often Very Difficult**
  - **Usually, *Lots* of Money Involved**
  - **Lives may be at Stake**
  - **Data may be Sparse**
- **When a Decision Analysis is Presented, Decision Makers Often**
  - **Ask Questions about Assumptions**
  - **Order Re-analysis with Different Assumptions**
  - **Generally Not Satisfied with Point Estimates**
  - **Afraid of Sparse Data**
- **Today will Share Experiences from Three Government Project Decisions for which *Clean Analyses* Led to *Quick Decisions***

Decision Problem #1:

NASA ISS O_2 Sensor Drift

- **On the International Space Station (ISS), The Extra-Vehicular-Activity (EVA) O_2 Sensor Measurements Drifting**
  - **Sensor Accuracy Requirement: ± 6mmHg for 270 Days post Calibration**
  - **Errors > 6mmHg: Astronaut May Suffer Bends during EVA**
  - **Errors < – 6mmHg: Astronaut May Suffer Oxygen Toxicity**
  - **Either may result in *Death* of Astronauts**
- **NASA Faced with Either**
  - **Halting ISS EVA's Until Sensor Redesign, Testing, and Deployment**
  - **Or, Compensating for the Error Drift to Reduce the Risk**
- **Drift Compensation Results were *not Convincing***

Decision Problem #2:
USCG C130 Cooling Turbine PM

- **Cooling Turbine Provides Cooling and Pressurization to the C130 Crew**
- **Failure in Service**
  - **Loss of Cooling, but More Important, Loss of Cabin Pressurization**
  - **Smoke, Loud, Crew Must Secure**
  - **Mission Compromised**
- **Costs**
  - **Replacement:** ***$30,000***
  - **Refurbishment:** ***$500***
- **Most Cost Effective Preventative Maintenance Interval?**

Decision Problem #3:
NASA ISS Bone Fracture Risks

- **On-Orbit Astronaut Bone Fractures could have Severe Consequences**
  - **To the Astronaut**
  - **To the Mission**
- **Very Low Probability Event – No Astronaut has Ever Broken a Bone during a Mission in History**
- **Risk Questions**
  - ***What is the Risk of Bone Fracture for Long Mars Missions?***
  - ***How Much will the Risk Increase if International Space Station Missions extend from 180 to 365 Days?***

Approach Used for These Decision Problems

- Find a Meaningful Decision Discriminator
  - Some Physical Quantity or Consequence
  - More Importantly, One the Decision Maker *Understands and will Use* to Make the Decision
- Gather the Available Data
- Use Bayesian Methods
  - Use a Most General Model for Data, Express Decision Discriminator in terms of Model Parameters
  - Use Non-Informative or Pseudo-Ignorance Priors
  - Formulate Joint Posterior Uncertainty Distribution for Model Parameters
- Sample Joint Posterior Uncertainty Distribution using *Markov Chain Monte Carlo* (*MCMC*) Methods

An Aside:
Markov Chain Monte Carlo

- **Just Like Ordinary Monte Carlo, Except for Sampling Approach**
  - **Ordinary Monte Carlo Uses *Built-in* Samplers for *Recognizable* Models**
    - **Usually Only Univariate Samplers Available**
    - **Possible Exception, Multivariate Normal**
  - ***MCMC* uses a Markov Chain to Sample a Density Function**
    - **Any Density Function – not Restricted to Recognized Built-in Models, *and Any Dimension!***
    - **Any Combination of Discrete and Continuous, One-sided, Two-sided, and Interval Random Variables**
- **All that is Required to Use *MCMC* is an Analytical Expression for the Density Function**

Continuing with the Approach

- Use *MCMC* Samples for the Joint Posterior Uncertainty Distribution for Parameters to Develop Samples of the Uncertainty Distribution for the Decision Discriminator, based on the Data
- Parameterize if Necessary
- Present *Uncertainty Distributions for the Decision Discriminator* for All Alternatives

Notes:
The Available Data

- Most of these Tough Decision Problems Have Few *if Any* Event Data
  - Risks of Financial Loss Should be Small
  - Risks of Failure Should be Very Small
  - Risks of Loss of Life Should be Tiny
- May have Plenty of *Censored* Data
  - Observations that Event or Loss has Not Occurred
  - Classical Statistical Approaches Almost Always Ignore
  - Resulting Bayesian Posterior Formulations Almost Always Analytically Intractable
- May have Outliers *– Or, Maybe Not*
  - May be Just one of those Rare Events
  - Should *never* Ignore Outliers

Notes:
Bayesian Methods

- **Decision Theory/Analysis has Long Historical Basis using Bayesian Methods**
- **Select the Most General Model Possible for Data**
  - **One-sided Data:** ***Weibull***
  - **Two-sided Data:** ***Non-central t***
  - **Interval Data:** ***Beta***

Avoids Some Assumptions

- **Use of Non-Informative, or Jeffreys', or Ignorance, or Reference Priors Obviates Questionable Assumptions**
  - **Usually Produces Analytically Intractable Joint Posterior**
  - **Forced to Use** ***MCMC***
  - **To Achieve Markov Chain Stability, Sometimes Must Wisely Truncate the Ignorance Prior –** ***Pseudo-Ignorance Prior***
- **Bounds Results Consistent with Information Theory**

Notes:

Using MCMC Joint Samples to Obtain Decision Discriminator Uncertainty Model Samples based on the Data

- **Fairly Simple Process: Evaluate Decision Discriminator at Joint MCMC Samples of Parameters**
- **What this Accomplishes**

$$pd\left(D \mid data\right) \propto \iiint\limits_{\substack{parameter\\domains}} D\left(\mid params\right) * pd\left(params \mid data\right) dparams$$

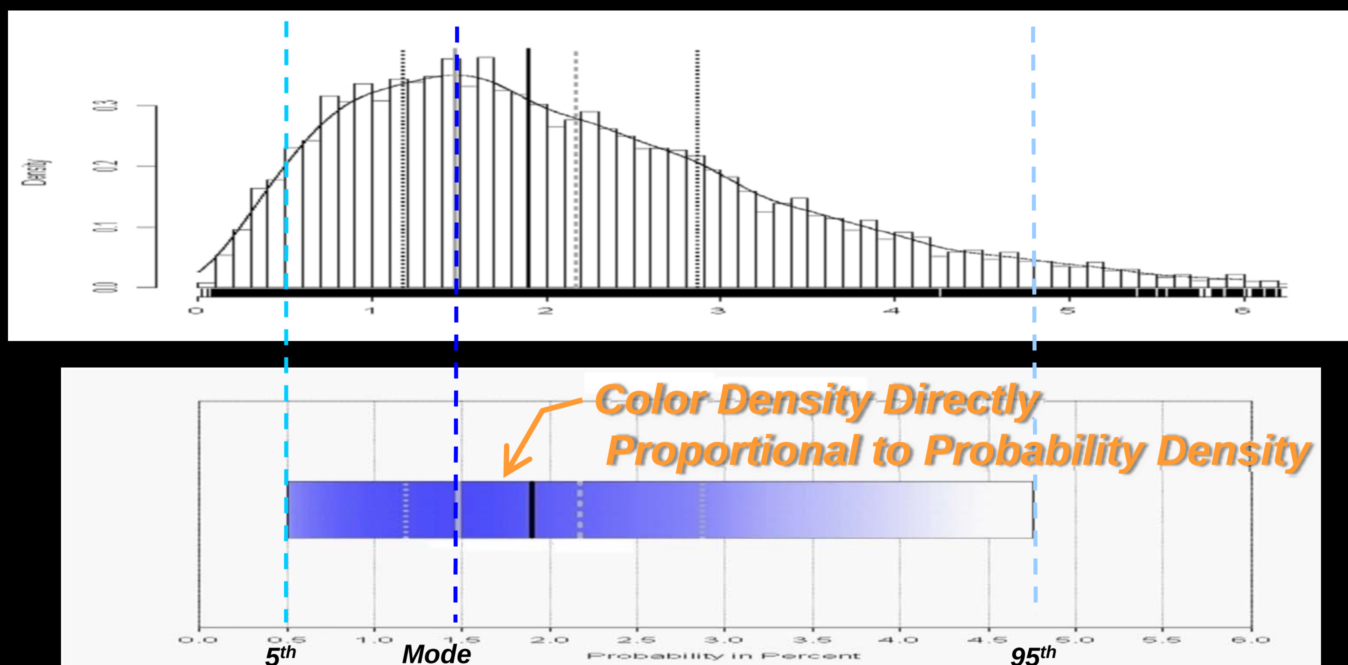
  - **Performs the Required Marginalization Integrals**
  - **Produces Samples of the Uncertainty Model for the Decision Discriminator**

If Needed, Parameterize

- **For Continuous Alternatives: Parameterize Decision Discriminator Uncertainty Distributions as Function of Alternative**
- **For Data with Covariates: Parameterize Decision Discriminator Uncertainty Distributions as Function of Covariates**
- **Simple, Merely Requires CPU Time**
- **Avoids a lot of Decision Maker *What if* Questions, as well as a lot of Analysis Repeats**

Alternative Distribution Presentations

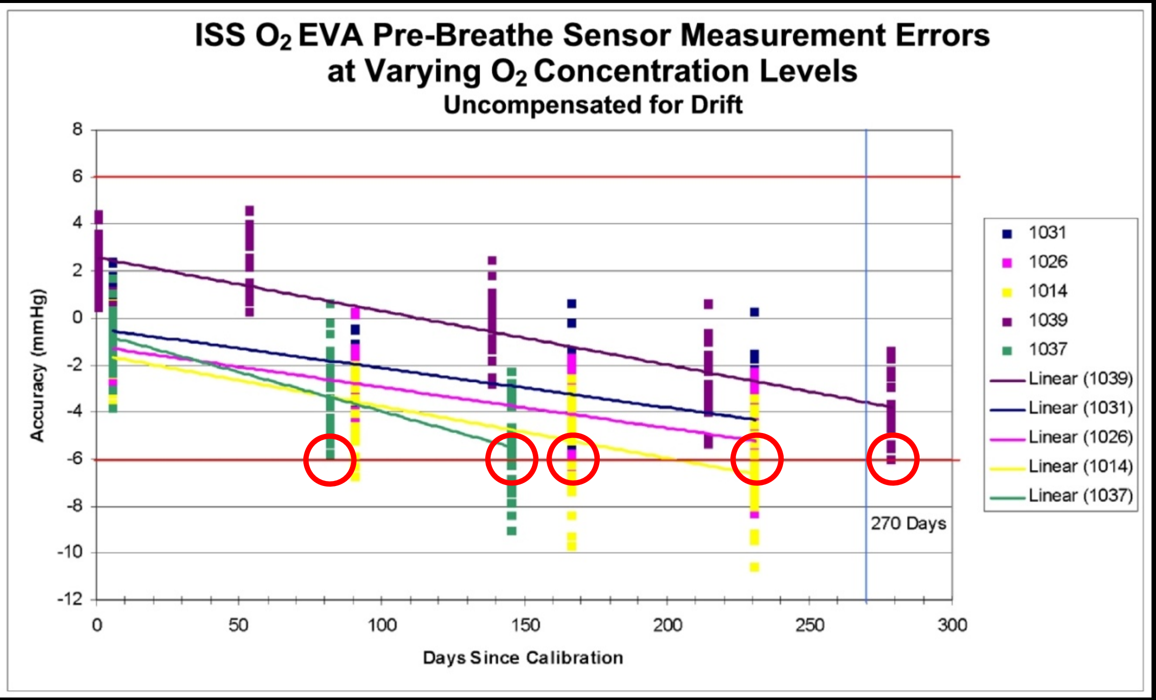
- **For Discrete Alternatives, Modified Bar Charts Work Well for Risk Comparisons**



Now, Decision Problem #1

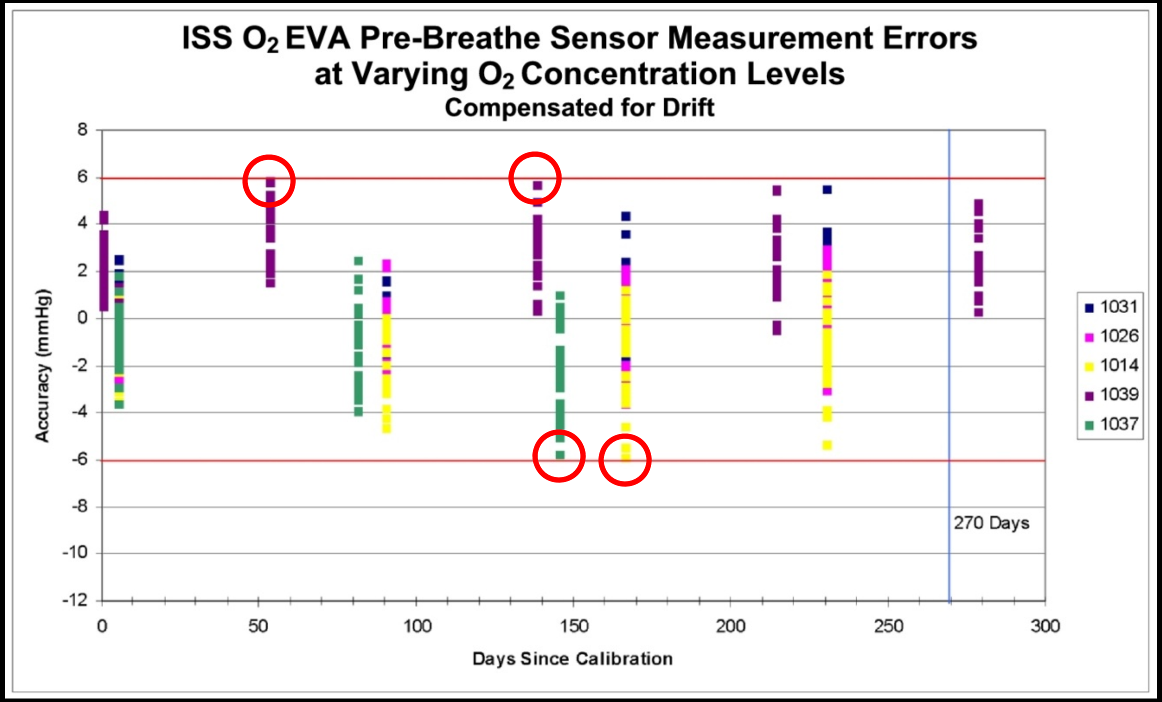
Observed ISS O_2 Sensor Errors

- Linear Least Squares Used to Look at Drift for Five Sensors
- All Appeared to Drift in Same Direction, with Similar Rates
- Compensation for Drift Might Reduce the Risk Enough



Compensation Scheme: *Use Least Squares on All Data to Estimate Slope and Intercept, and Remove from Sensor Measurements*

Sensor Errors After Drift Compensation



- Unacceptable Drift Errors Occur *Even Earlier!*
- Did the Risk *Actually Increase?*
- What was the Risk *without* Drift Compensation?
- No Answers, *No Decision!*

Decision Analysis

- **Decision Discriminator: Risk of Exceeding e_{max} (±6mmHg) at TSC = 270 days** $R\left(\left|e_s\right| > e_{max} \mid 270, \mu_0, \mu', \sigma_s\right) = 2 * \Phi\left(-e_{\max} \mid \mu_0 + \mu' * 270, \sigma_s\right)$
- **Data: Preceding Slides, Before and After Compensation**
- **Bayesian Approach**
  - **Used Normal Model with Covariate for TSC since Linear Regression was Used to Compute Drift Correction Parameters**
  - **Joint Posterior with Ignorance Priors *NOT* Analytically Tractable, Used *MCMC* Sampling**
- **Decision Discriminator Uncertainty Model Transform**

$pd\left(R\left(\left|e_s\right| > e_{max} \mid 270\right) \mid data\right)$

$\propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \, d\mu_0\, d\mu'\, d\sigma_s$

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Not A… …able!

Won't Need It!

- **Use Modified Bar Charts before and after Compensation**

Another Aside:
Decision Discriminator Uncertainty

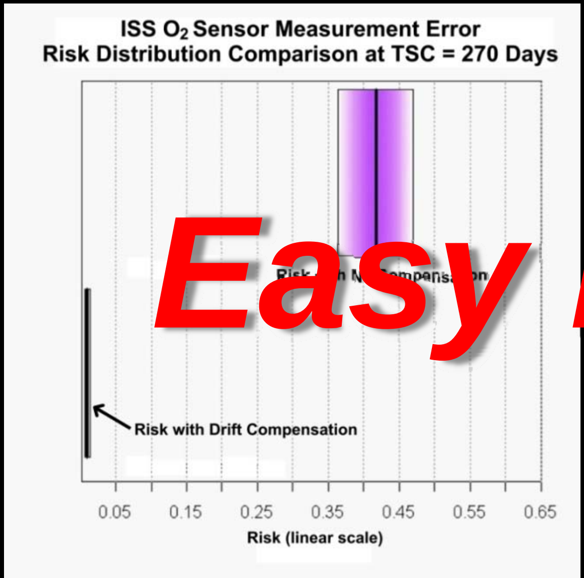
- **Obtain Samples by Simply Evaluating Decision Discriminator Equation at Joint Samples of Posterior**
- **Suppose Want to Know Assurance *Based on the Data* that Risk of Exceeding e_{max} at TSC = 270 days is Less than 5%**
  - **Have *M* Joint Posterior Samples from MCMC**
  - **Evaluate Decision Discriminator Equation at Each Joint Sample at TSC = 270 days – Get *M* Samples of Risk of Exceeding e_{max} at TSC = 270 days Based on the Data (for Modified Bar Charts)**
  - **Count Number of Risk Samples < 0.05 and divide by *M***

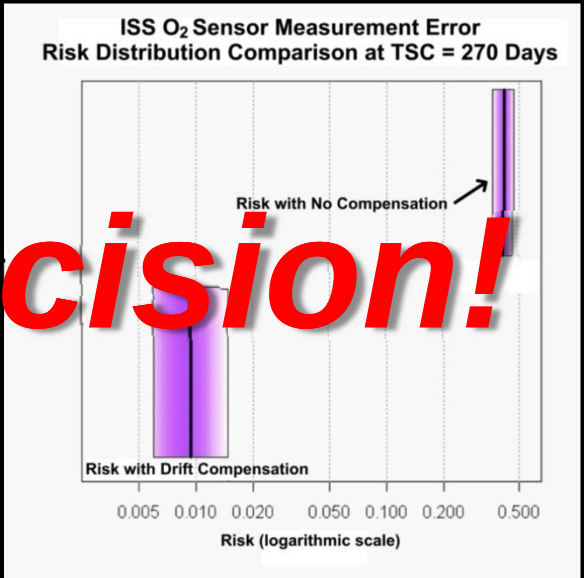
$$P\left(R\left(\left|e_s\right| > e_{max} \mid \mathrm{TSC} = 270\right) < 0.05 \mid data\right)$$

$$= \frac{\sum_{i=1}^{M} \begin{bmatrix} 1 \mid 2 * \Phi\left(-e_{max} \mid \mu_{0i} + \mu_i' * 270, \sigma_{si}\right) < 0.05 \\ 0 \mid 2 * \Phi\left(-e_{max} \mid \mu_{0i} + \mu_i' * 270, \sigma_{si}\right) \geq 0.05 \end{bmatrix}}{M}$$

Risk Assessment Results

- Obtained 10,000 Joint MCMC Samples *of* μ_0, μ', and σ_s for Covariate Data With and Without Drift Compensation
- Used to Obtain Risk Samples for both at TSC = 270 days





Easy Decision!

- 90% Certain Based on the Data, Risk of Exceeding e_{max} without Drift Compensation within 270 Days Between 36% and 46%
- 95% Certain Based on the Data, Risk of Exceeding e_{max} with Drift Compensation within 270 Days is less than 1.5%

Decision Problem #2

USCG C130 Cooling Turbine PM

- 60:1 Cost Ratio, Replace:Maintain
- Only Had Five Failure Data: 463, 538, 1652, 1673, and 2462 flight hours
- Only Had One Survivor Datum: 96 flight hours
- What PM Interval to Select?
- USCG Decision Makers *Paralyzed*

Decision Analysis

- **Decision Discriminator: *CS_{tpm}* – Cost Savings per Flight Hour in performing Preventative Maintenance at the Interval of *t_{pm}* flight hours over Allowing Failures in Service**

$$CS_{tpm} = \left(\frac{C_{rep}}{\eta}\right)\gamma\left(\frac{\beta-1}{\beta}, \left(\frac{t_{pm}}{\eta}\right)^{\beta}\right) - \left(\frac{C_{pm}}{t_{pm}}\right) * e^{-\left(\frac{t_{pm}}{\eta}\right)^{\beta}}$$

- **Data: Preceding Slides, 5 Failures Events, One Survivor**
- **Bayesian Approach**
  - **Used Weibull Model**
  - **Posterior with Ignorance Priors *NOT* Analytically Tractable, Use *MCMC* Sampling**
- **Decision Discriminator Uncertainty Model Transform**

$pd(CS_{tpm}$ …

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Not A… …able!

Won’t Need It!

- **Use Parameterization as a Function of *t_{pm}***

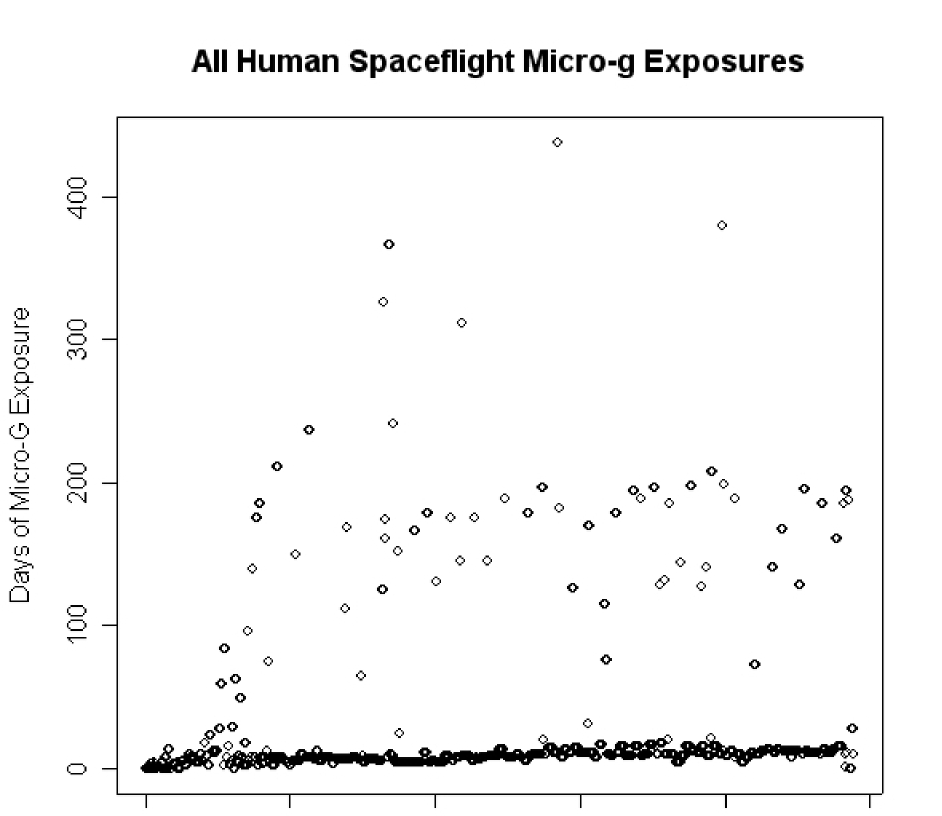
Cost Savings Risks Using a PM Interval - Parameterized

- Full Distributions Per Flight Hour Based Solely on The Data, Per Bird – CS_{tpm}
- Obtained by Evaluating CS_{tpm} at the Joint Posterior MCMC Samples Parameterized as a function of PM Interval in flight hours
- Plotted Only 5th, Most Likely, and 95th percentile Cost Savings Risks
- At t_{pm} = 250 hours, 95% Certain, based on the data, that USCG can ***SAVE*** at least $17 per flight hour per bird

Decision Problem #3

Available NASA Bone Fracture Data

- 977 Astronaut Missions of Varying Lengths (as of May 2005)
- No Events Observed
  - No Bones Broken
  - Did Observe 977 Mission Lengths without a Broken Bone



Decision Analysis

- **Decision Discriminator:** ***Risk of Bone Fracture*** $R_{T_M} = 1 - e^{-\left(\frac{T_M}{\eta}\right)^{\beta}}$
- **Data: Preceding Slides, 977 Censored Data**
- **Bayesian Approach**
  - **Used Weibull Model**
  - **Posterior with Ignorance Priors *NOT* Analytically Tractable, Use *MCMC* Sampling**
- **Decision Discriminator Uncertainty Model Transform**

$pd\left(R_{T_M} \ldots d\eta\, d\beta\right.$

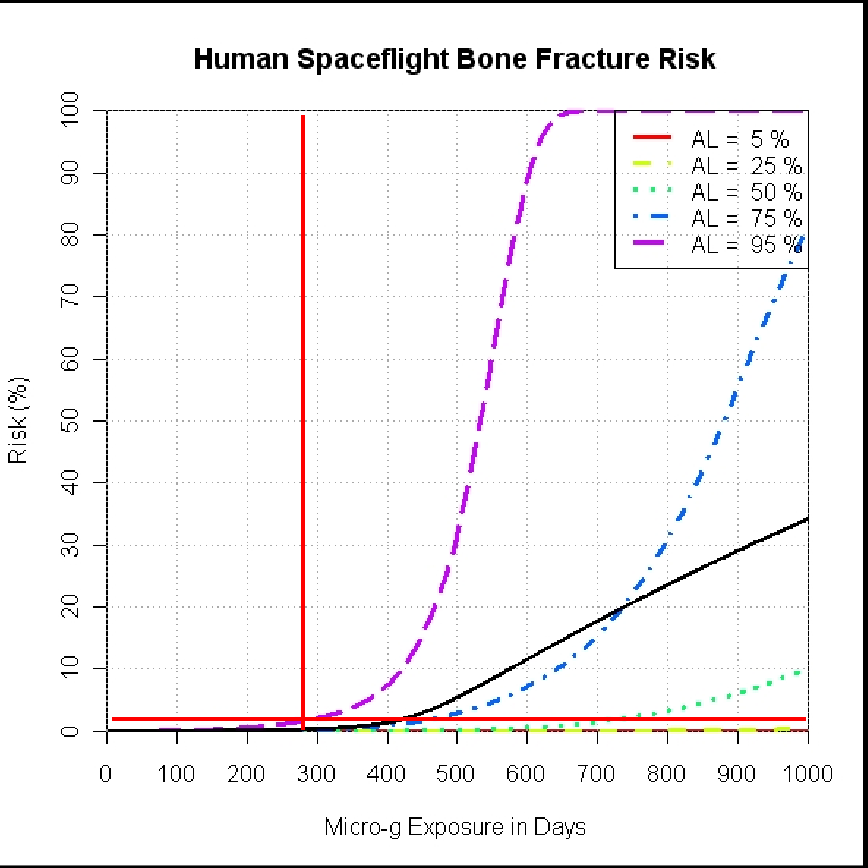
A Ve… …on,
Not A… …able!

Won't Need It!

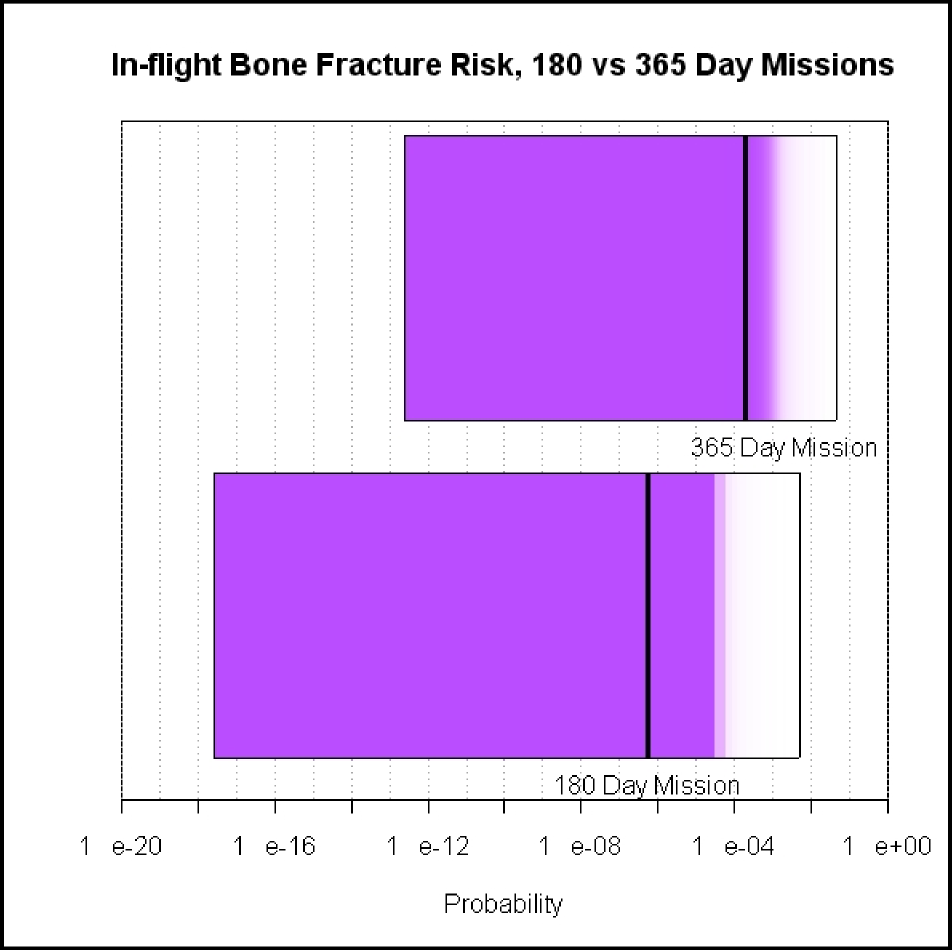
- **Use Parameterization and Modified Barcharts**

Parameterized Risk Results

- **Risk Uncertainty Distribution Parameterized As a Function of Mission Duration Obtained by Evaluating Risk Equation at MCMC Samples**
- **Parameterized and Plotted Various Assurance Levels (5, 25, 50, 75, 95%)**
- **For Mars Missions of 270 Days – We Can be 95% Certain that Risk of fracture during the Mission is < 3%, *Based on the Information Available***



The ISS Mission Extension Question



Summary

- **The Decision Analyses Used were *Clean***
  - **Selected Meaningful and Useful Decision Discriminators**
  - **Using Ignorance or Pseudo-Ignorance Priors Limited Use of Questionable Assumptions**
  - **Used Parameterizations**
  - **Presented Uncertainty Distributions for Decision Discriminators for All Alternatives, based on the Data**
- **Decisions Made *Almost Immediately* for All Three Examples**
  - **Decision Makers were Comfortable Deciding**
  - **Saved Money in All Cases**

Conclusion

- **Have Published Papers for these 3 Examples, Contact me and I will Share**
- **Available to Help with *Tough* Decision Analysis Problems**
- **Or, I Can Teach Your Folks How to Perform Clean Decision Analysis and Achieve Quick Decision**
- ***Link with me*: http://www.linkedin.com/in/attwatermarkpowell**

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